8Ratio and percentages

1 Ratio

A ratio is a comparison of two numbers. We generally separate the two numbers in the ratio with a colon (:) or as a fraction.

Suppose we want to write the ratio of 8 and 12.

We can write this as 8:12 or as 8/12, and we say the ratio is *eight to twelve*.

Examples:

Janet has a bag with 4 pens, 3 sweets, 7 books, and 2 sandwiches.

1. What is the ratio of books to pens?

Expressed as a fraction, the answer would be 7/4.
Two other ways of writing the ratio are 7 to 4, and 7:4.

2. What is the ratio of sweets to the total number of items in the bag?

There are 3 candies, and 4 + 3 + 7 + 2 = 16 items total.
The answer can be expressed as 3/16, 3 to 16, or 3:16.

2 Comparing Ratios

To compare ratios, write them as fractions. The ratios are equal if they are equal when written as fractions.

We can find ratios equivalent to other ratios by multiplying/ dividing both sides by the same number.

Example:

Are the ratios 2 to 7 and 4:14 equal?

The ratios are equal because 2/7 = 4/14.

The process of finding the simplest form of a ratio is the same as the process of finding the simplest form of a fraction.
1 : 3.5 or 2 : 7 could be given as more simple forms of the ratio 4 : 14.

Exercise 1 Simplify the following ratios:

3:6

25:50

40:100

9:21

11:121

For some purposes the best is to reduce the numbers to the form 1 : n or n : 1 by dividing both numbers by either the left hand side or the right-hand side number. It is useful to be able to find both forms, as any of them can be used as the unit in a problem.

Examples:

Which will be the divisor if we are to reach the form 1 : n for the ratio 4 : 5 ?
The divisor will be 4 and the ratio will be 1 : 1.25

And if we are to reach the form n : 1 for the same ratio?
The divisor will be 5, and the ratio will be 0.8 : 1

Exercise 2 Reduce to the form 1:n and n:1 10 shovels of cement, 25 shovels of sand

Exercise 3 Reduce to the form 1:n the following:

4:9

7:5

30:100

5:12

Exercise 4 Reduce to the form n:1 the following:

14:9

15:5

40:100

75:12

3. Direct Proportionality

We say that there is a direct proportionality between two magnitudes if when we double one magnitude, the other also doubles, when we half the first, the second also halves.

Examples:
If a can of cola costs 40p, the cost of:
2 cans is 80p
5 cans is £2.00

We can see that, for example, if we double the number of cans, we double the price. We say that the total cost of the cans increases proportionally with their number.

A proportion is one equality with a ratio on each side. It is a statement that two ratios are equal.

3/4 = 6/8 is an example of a proportion.

Note that proportions, ratios and equalities with fractions are different forms of expressing the same idea.

When one of the four numbers in a proportion is unknown, cross products may be used to find the unknown number. This is called solving the proportion. Letters are frequently used in place of the unknown number.

Example:

Solve for n: $1/2 = n/4$.
Using cross products we see that $2 \times n = 1 \times 4 = 4$, so $2 \times n = 4$. Dividing both sides by 2, $n = 4 \div 2$ so $n = 2$.

Exercise 5 Find the unknown side in each ratio or proportion:

1:7 = 5:x

1/9 = 5/x

1:5 = 3:x

2/7 = x/21

3:4 = 7:x

5/8 = 8/x

1/4=3/x

2 : 3 = 5 : x

12 : 21 = 16 : x

Exercise 6 Think of examples of proportionality in real life

Exercise 7

7.1 If 3 litres of petrol cost 3.45 €. How much will cost

   a) 5 litres

   b) 23.5 litres

7.2 If we travel 136 km in 1.5 hours driving at a constant speed.

    a) How many km will we travel in 7.4 h?

    b) How many hours do we need to travel 200 km?

7.3 Adrian finds that in each delivery of 500 bricks there are 20 broken bricks. How many bricks are broken in a delivery of 7500?

7.4 In a drink 53 ml of fruit juice are mixed with 250 ml of water. How many litres of water there are in 30l of that drink?

7.5 A car uses 25 litres of petrol to travel 176 miles. How far will the car travel using 44 litres of petrol?

4. Inverse Proportionality

Look at the relationship that exists between the number of the members of a family and the days that one box of apples lasts them (suppose that all the people eat the same amount of apples at the same rate).

Observe that the more people there are in the family the less time the box of fruit lasts, and the less people there are, the longer it lasts.

We will say that this relationship is of inverse proportionality

> We say that there is an inverse proportionality between two magnitudes if when increasing one magnitude, (double, triple…) the other decreases (half, third…), when decreasing one (half, third…), the other increases (double, triple…).
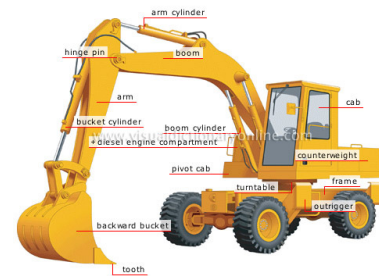
Exercises 8

8.1 A truck that carries 3 tons need 15 trips to carry a certain amount of sand. How many trips are needed to carry the same amount of sand with another truck that carries 5 tons?

8.2 An automobile factory produces 8100 vehicles in 60 days. With the production rhythm unchanged. How many units will be made in one year?

8.3 A driver takes $3\dfrac{1}{2}$ hours to drive 329 km. How long will it take another trip in similar conditions as the previous one, but travelling 282 km instead?

8.4 Two hydraulic shovels make the trench for a telephone cable in ten days. How long will it take to make the trench with 5 shovels?

5. Ratios with more than two parts

There are ratios with 3 or more parts.

Example: Tim, Tom and Tam are brothers. Tim has £10, Tom has £20 and Tam has £30. The money they have as a ratio is 10 : 20 : 30

We can simplify to 1 : 2 : 3

Exercise 9 Three angles are *a* = 60°, b = 80°, c = 80°. Write them as a ratio, and then simplify the ratio.

Can these be the angles of a triangle? If the answer is no, calculate the angles of a triangle with this ratio.

Exercise 10 The lengths of the sides of a quadrilateral are 4 cm, 4 cm, 6 cm and 6 cm. Find their ratio and simplify it.

T: What type of quadrilateral can it be if the sides are listed in order?

Exercise 11 Red blue and yellow paint are mixed in the ratio 3:2:7 to produce 6 litres of another colour. How much of each colour paint is used?

6. Percentages

A percent is a ratio of a number to 100. A percent is expressed using the symbol %.
A percent is also equivalent to a fraction with denominator 100.

Examples:

a) 5% of something = 5/100 of that thing.

b) 52% = 52/100 = 13/25 (nearly equals 1/2)

c) 8/200 is what %?

Method 1: 8/200 = (4 × 2)/(100 × 2), so 8/200 = 4/100 = 4%.

Method 2: Let 8/200 be n%. n = (8x100):200 = 4, so n% = 4%.

6.1 Percent as a decimal

Percent and hundredths are basically equivalent. This makes conversion between percent and decimals very easy.

To convert from a decimal to a percent, just move the decimal 2 places to the right.

Examples:

0.15 = 15 hundredths = 15%.

0.0006 = 0.06%

Converting from percent to decimal form is similar, only you move the decimal point 2 places to the left.

Examples:

Express 3% in decimal form.

Moving the decimal 2 to the left (and adding in 0's to the left of the 3 as place holders,) we get 0.03.

Express 97'5% in decimal form. We move the decimal 2 places to the left to get 0.975.

Exercise 12 Convert the following percentages to decimals.

    a)   75%   b) 1.74%

    c)   3.7%   d) 80%

    e)   15%   f)   0.6%

    g) 9%   h) 0.07%

Exercise 13 Convert the following decimals to percentages.

    a)   0.5   b) 0.74

    c) 0.35   d) 0.08

    e)   0.1   f) 0.52

    g)   0.8   h) 0.07

i)   0.04   j)   0.18

k)   0.4   l)   0.3

6.2 Common percentages

Some percentages are very common and it is useful to know them as fractions or decimals.

Some very simple percents are:

100% = 1   50% = 1/2 = 0.5   25% = 1/4 = 0.25

10% = 1/10 = 0.1   1% = 1/100 = 0.01

7 Calculations with percentages

7.1 Percentage of a quantity

To calculate the percentage of a quantity we must multiply it by the percent and divide by 100.

Example:

Calculate the 5% of 72

5% of 72 is $\dfrac{72 \cdot 5}{100} = 3.6$

Exercise 14 Calculate:

a) 15% of 540   **b) 3% of 320**

c) 5.3% of 7   **d) 6% of 5430**

7.2 Calculate the number when we know the Percentage

We must multiply the percentage by 100 and divide by the percent.

Example:

The 22% of a number is 66, which is the number?

The quantity is $\dfrac{66 \cdot 100}{22} = 300$

Exercise 15 Calculate the number if:

a) 95% of the number is 1020

b) 5% of the number is 7

c) 15.3% of the number is 500

d) 83% of the number is 1086

7.3 Express a ratio or a fraction as a percent

To write as a percent a ratio or a fraction , we may convert them into a number dividing and then multiply by 100

Example:

Calculate 27 out of 32 as a . We do $\dfrac{27}{32} \cdot 100 = 84.3$ %

Exercise 16 Express each of the following as percentages.

a) 8 out of 50

b) 3 out of 25

c) 8 out of 20

d) 3 out of 10

e) 6 out of 50

f) 6 out of 40

g) 12 out of 80

h) 84 out of 200

i) $\dfrac{23}{75}$

j) $\dfrac{3}{5}$

k) $\dfrac{102}{37}$

7.4 Estimating percentages

Estimating a percent is as easy as estimating a fraction as a decimal, and converting to a percent by multiplying by 100.

To estimate the percent of a number, we may convert the percent to a fraction, if useful, to estimate the percent.

Example:

Estimate 19 as a percent of 80.

As a fraction, 19/80 $\cong$ 20/80 = 1/4 =0.25 = 25%.

The exact percent is in fact 23.75%, so the estimate of 25% is only 1.25% off. (About 1 part out of 100.)

Exercise 17 Estimate, and later calculate the exact answer:

7 as a percent of 960.

12% of 72.

9.6% of 51.

7.6 Calculate a number increased with a percentage

We add to the number the percentage.

Example: Calculate the value of 320 increased by a 5%

1. The increase is 5% of 320 is $\dfrac{320 \cdot 5}{100} = 16$

2. The final value is 320 + 16 = 336

7.7 Calculate a number decreased with a percentage

We subtract to the number the percentage.

Example: Calculate the value of 320 decreased by a 12%

1. The decrease is 12% of 320 is $\dfrac{320 \cdot 12}{100} = 38.4$

2. The final value is 320 − 38.4 = 281.6

7.7 Percentage increase decrease

> Percent increase and decrease of a value measure how that value changes, as a percentage of its original value.

Percentage *increases and decreases* are calculated using:

$$\text{Percentage increase} = \frac{\text{increase}}{\text{initial value}} \, \text{x100}$$

$$\text{Percentage decrease} = \frac{\text{decrease}}{\text{initial value}} \times 100$$

Examples:

1. A collectors' comic book is worth 120 € in 2004, and in 2005 its value is 132 €. The increase in price is 12€; 12 is 10% of 120, so we say its value increased by 10%.

2. Amy is training for the 1500 meter run. When she started training she could run 1500 meters in 5 minutes and 50 seconds. After a year of practice her time decreased by 8%. How fast can she run the race now?

Her old time was 5 × 60 + 50 = 350 seconds, and 8% of 350 are 28, so she can run the race in 350 - 28 = 322 seconds (5 minutes and 22 seconds).

3. A compact disc that sells for $12 is on sale at a 20% discount. How much does the disc cost on sale? The amount of the discount is 20% of $12, which is $2.40, so the sale price is $12.00 - $2.40 = $9.60.

Exercises 18

18.1 A magazine for teenagers sells 110000 copies each month. The company's president wants to increase the sales by 6%. How many extra magazines would they have to sell to reach this goal?

18.2 Chocolate bars normally cost 80 cents each, but are on sale for 40 cents each. What percent discount is this?

18.3 Movie tickets sell for $8.00 each, but if you buy 4 or more you get $1.00 off each ticket. What percent discount is this?

18.4 A firm decides to give 20% extra free in their packets of soap powder. How much extra soap powder would be given away free with packets which normally contain

    (a) 2 kg of powder

    (b) 1.2 kg of powder?

18.5 A house costs £30,000. A buyer is given a 10% discount. How much money does the buyer save?

18.6 Karen bought an antique vase for £120. Two years later its value increases by 25%. What is the new value of the vase?

18.7 When Wendy walks to school she covers a distance of 1800 m. One day she discovers a shortcut, which reduces this distance by 20%. How much shorter is the new route?

18.8 Kim's mother decides to increase her pocket money by 40%. How much extra does Kim receive each week if previously she was given £2.00 per week?

18.9 Express '30 out of 40' and '40 out of 50' as percentages. Which is the best score?

EXTRA EXERCISES

1. The price of a bar of chocolate is 35 c and includes 7 c profit. Express the profit as a percentage of the price.

2. In the crowd at a football match there were 35000 *Manchester United* supporters and 25000 *Manchester City* supporters. What percentage of the crowd supported each team?

3. A school won a prize of £1800. The Headmaster spent £750 on a new computer and the rest on software. What percentage of the money was spent on software?

4. James needs another 40 football stickers to complete his collection. There are a total of 500 stickers in the collection. What percentage of the collection does he have?

5. A 750 ml bottle of shampoo contains 200 ml of free shampoo. What percentage is free?

6. Adrian finds that in a delivery of 1500 bricks there are 50 broken. What is the percentage of broken bricks?

7. A glass of drink contains 50 ml of fruit juice and 200 ml of lemonade. What percentage of the drink is fruit juice?

8. Find each of the following, giving your answers to the nearest penny.
      a) 30% of £150   **b) 12% of £903**   **c) 12.6% of £140**
      d) 4.5% of £320   **e) 5.9% of £50**   **f) 8.2% of £18.**

9. A MP4 player has a normal price of £150.

  a) In a sale its normal price is reduced by 12%. Find the sale price.
  b) After the sale, normal prices are increased by 12%. Find the new price of the MP4 player.

10. Peter earns £9000 per year. He does not pay taxes on the first £3500 he earns and pays 25% taxes on the rest. How much tax does he pay?

11. A new gas supplier offers a 25% discount on the normal price and a further 5% discount if customers pay on line. For one client the gas bill is £130. Find out how much they have to pay after both discounts.

Remember

$$\text{Percentage increase} = \frac{\text{increase}}{\text{initial value}} \times 100$$

$$\text{Percentage decrease} = \frac{\text{decrease}}{\text{initial value}} \times 100$$

12. A baby weighed 5.6 kg and six weeks later her weight increases to 6.8kg. Find the percentage increase.

13. A factory produces blank DVDs at a cost of 88p and sells them for £1.10. Find the percentage profit.

14. A new car cost £11500 and one year later it is sold for £9995. Find the percentage reduction in the value of the car.

15. An investor bought some shares at a price of £14.88 each. The price of the shares drops to £11.45. Find the percentage loss.

16. A supermarket offers a £8 discount to all customers spending more than £40. Karen spends £42.63 and John spends £78.82. Find the percentage saving for each one.

17. In a year, the value of a house increases from £146 000 to £148 000. Find the percentage increase in the value of the house.

18. A battery powers an iPod for 12 hours. An improved version of the battery powers the iPod for an extra 30 minutes. Find the percentage increase in the life of the batteries.

Solutions

Exercise 1 1:2, 1:2, 2:5, 3:7, 1:11.

Exercise 2 1:2.5 and 0.4:1

Exercise 3 1:2.25, 1:0.71, 1:3.$\overset{\frown}{3}$, 1:2.4.

Exercise 4 1.$\overset{\frown}{5}$:1, 3:1, 0.4:1, 6.25:1

Exercise 5 $x = 35$, $x = 45$, $x = 15$, $x = 6$, $x = 9.\overset{\frown}{3}$, $x = 12.8$, $x = 12$, $x = 7.5$, $x = 28$.

Exercise 6

Exercise 7

7.1 a) 5.75 €, b) 27.03 €. **7.2** a) 670.93 km, b) 2.2h or 2h 12 minutes. **7.3** 300 bricks. **7.4 2**4.75 l **7.5** 309.76 miles.

Exercises 8

8.1 25 trips. **8.2** 49275 vehicles. **8.3** 3 h. **8.4** 25 days.

Exercise 9 60:80:80; 3:4:4 or 0.75:1:1;no, they can not be the angles of a triangle; 49.09º, 65.$\overset{\frown}{45}$º and 65.$\overset{\frown}{45}$º

Exercise 10 4:4:6:6, 1:1:1.5:1.5 T: It is a trapezoid (kite).

Exercise 11 1.5 of red, 1 l of blue and 3.5 l of yellow paint.

Exercise 12 a) 0.75, b) 0.0174, c) 0.037, d) 0.8, e) 0.15, f) 0.006, g) 0.09, h) 0.0007.

Exercise 13 a) 50%, b) 74%, c) 35%, d) 8%, e) 10%, f) 52%, g) 80%, h) 7%, i) 4%, j) 18%, k) 40%, l) 30%.

Exercise 14 a) 81, b) 9.6, c) 0.371, d) 325.8.

Exercise 15 a) 1073.68, b) 140, c) 3267.97, d) 1308.43.

Exercise 16 a) 16%, b) 12%, c) 40%, d) 30%, e) 12%, f) 15%, g) 15%, h) 42%, i) 30.66%, j) 60%, k) 275.68%.

Exercise 17 7% and 0.729%, 8% and 8.64%, 5% and 4.896%

Exercises 18

18.1 6600 magazines. **18.2** 50%. **18.3** 12% discount in each extra ticket. **18.4** (a) 0.4kg, (b) 0.24 kg. **18.5** £3000. **18.6** £150. **18.7** 360m. **18.8** 80 p. **18.9** 75% and 80%, '40 out of 50' is the best score.

EXTRA EXERCISES

1. 20%. **2.** 58.3% *Manchester United* supporters and 41.7% *Manchester City* supporters. **3.** 58.3%. **4.** 92%. **5.** 26.6%. **6.** 3.3%. **7.** 20%. **8.** a) £45, b) £108.30, c) £17.64, d) £14.40, e) £2.95, f) £1.48. **9.** a) £132, b) £168.
10. 1375£. **11.** 92.63£. **12.** 21.4%. **13.** 25%. **14.** 13%. **15.** 23%. **16.** Karen 18.77% and John 10%. **17.** 1.4%. **18.** 4.2%